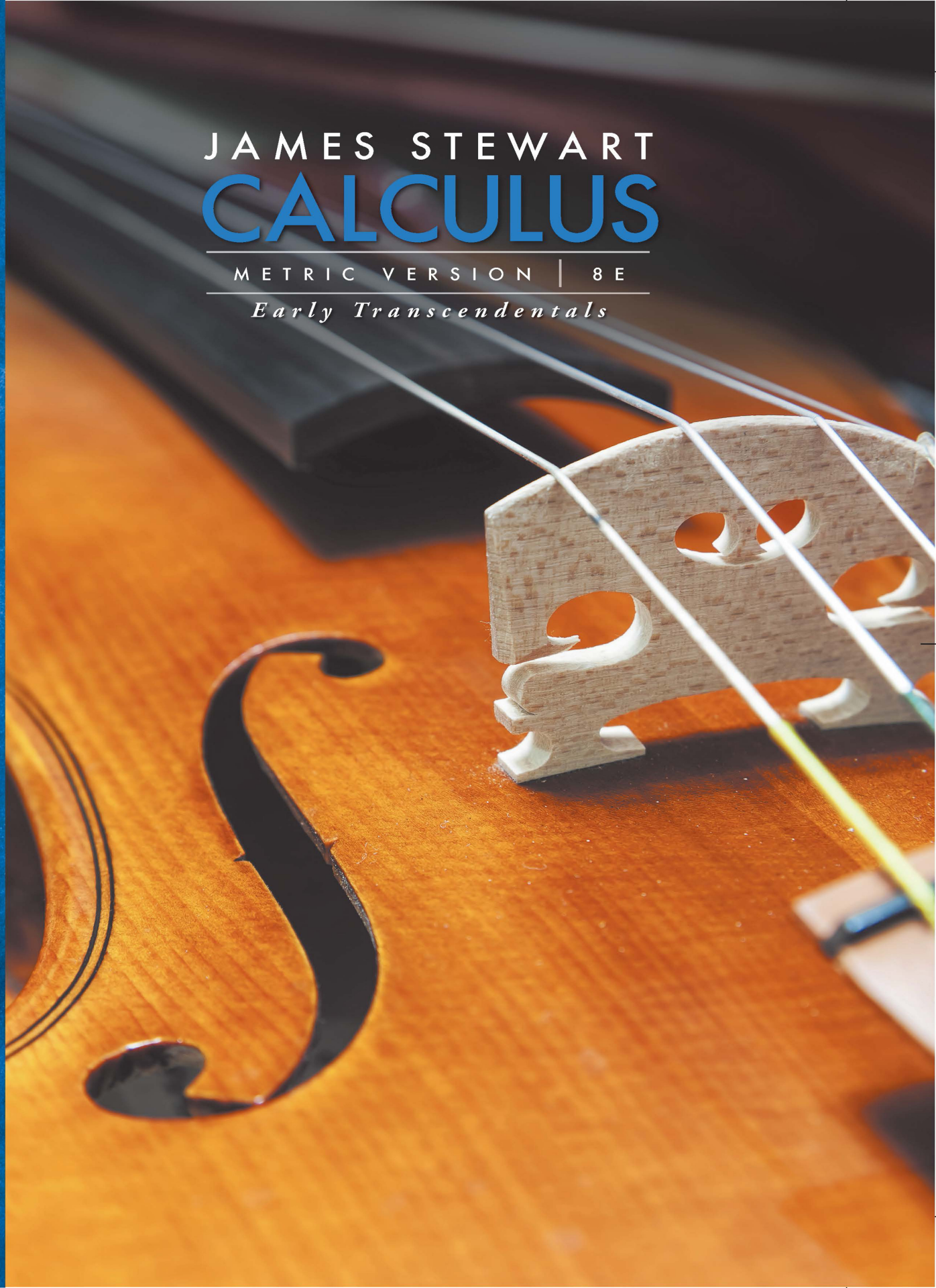
METRIC VERSION



JAMES STEWART

CALCULUS

METRIC VERSION | 8E

Early Transcendentals

CALCULUS

EARLY TRANSCENDENTALS

EIGHTH EDITION

METRIC VERSION

JAMES STEWART

McMASTER UNIVERSITY

AND

UNIVERSITY OF TORONTO

CENGAGE Learning®

Australia • Brazil • Mexico • Singapore • United Kingdom • United States

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

This is an electronic version of the print textbook. Due to electronic rights restrictions, some third party content may be suppressed. Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. The publisher reserves the right to remove content from this title at any time if subsequent rights restrictions require it. For valuable information on pricing, previous editions, changes to current editions, and alternate formats, please visit www.cengage.com/highered to search by ISBN#, author, title, or keyword for materials in your areas of interest.

Important Notice: Media content referenced within the product description or the product text may not be available in the eBook version.

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s). Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

CENGAGE Learning®

***Calculus: Early Transcendentals*, Eighth Edition, International Metric Version**
James Stewart

Product Manager: Gary Whalen
Senior Content Developer: Stacy Green
Associate Content Developer: Samantha Lugtu
Product Assistant: Stephanie Kreuz
Media Developer: Lynh Pham
Marketing Manager: Ryan Ahern
Content Project Manager: Cheryll Linthicum
Art Director: Vernon Boes
Manufacturing Planner: Becky Cross
Production Service: TECHarts
Photo and Text Researcher: Lumina Datamatics
Copy Editor: Kathi Townes, TECHarts
Illustrator: TECHarts
Text Designer: Diane Beasley
Cover Designer: Denise Davidson
Compositor: Stephanie Kuhns and
Kristina Elliott, TECHarts
Cover Image: iStock/Getty Images Plus

© 2016, 2012 Cengage Learning

WCN: 02-300

ALL RIGHTS RESERVED. No part of this work covered by the copyright herein may be reproduced, transmitted, stored, or used in any form or by any means graphic, electronic, or mechanical, including but not limited to photocopying, recording, scanning, digitizing, taping, Web distribution, information networks, or information storage and retrieval systems, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without the prior written permission of the publisher.

For permission to use material from this text or product, submit all requests online at **www.cengage.com/permissions**.

Further permissions questions can be e-mailed to **permissionrequest@cengage.com**.

Library of Congress Control Number: 2014951195

ISBN: 978-1-305-27237-8

Cengage Learning Global Offices

Asia
www.cengageasia.com
tel: (65) 6410 1200

Australia / New Zealand
www.cengage.com.au
tel: (61) 3 9685 4111

Brazil
www.cengage.com.br
tel: (55) 11 3665 9900

India
www.cengage.co.in
tel: (91) 11 4364 1111

Latin America
www.cengage.com.mx
tel: (52) 55 1500 6000

UK / Europe / Middle East / Africa
www.cengage.co.uk
tel: (44) 0 1264 332 424

Cengage Learning is a leading provider of customized learning solutions with office locations around the globe, including Singapore, the United Kingdom, Australia, Mexico, Brazil, and Japan. Locate your local office at **www.cengage.com**.

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

To learn more about Cengage Learning Solutions, visit **www.cengage.com**. **Purchase any of our products at your local college store or at our preferred online store www.cengagebrain.com.**

Windows is a registered trademark of the Microsoft Corporation and used herein under license.
Macintosh is a registered trademark of Apple Computer, Inc. Used herein under license.
Maple is a registered trademark of Waterloo Maple, Inc.
Mathematica is a registered trademark of Wolfram Research, Inc.
Tools for Enriching Calculus is a trademark used herein under license.

K07T15

Printed in the United States of America
Print Number: 01 Print Year: 2015

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s). Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Contents

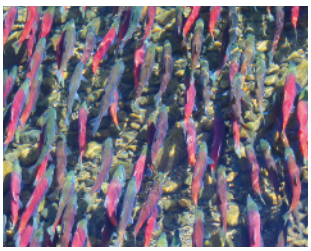
PREFACE xi
TO THE STUDENT xxiii
CALCULATORS, COMPUTERS, AND OTHER GRAPHING DEVICES xxiv
DIAGNOSTIC TESTS xxvi

A Preview of Calculus 1

1 Functions and Models 9

1.1 Four Ways to Represent a Function 10
1.2 Mathematical Models: A Catalog of Essential Functions 23
1.3 New Functions from Old Functions 36
1.4 Exponential Functions 45
1.5 Inverse Functions and Logarithms 55
Review 68
Principles of Problem Solving 71

2 Limits and Derivatives 77



2.1 The Tangent and Velocity Problems 78
2.2 The Limit of a Function 83
2.3 Calculating Limits Using the Limit Laws 95
2.4 The Precise Definition of a Limit 104
2.5 Continuity 114
2.6 Limits at Infinity; Horizontal Asymptotes 126
2.7 Derivatives and Rates of Change 140
Writing Project • Early Methods for Finding Tangents 152
2.8 The Derivative as a Function 152
Review 165
Problems Plus 169

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

3 Differentiation Rules 171

3.1 Derivatives of Polynomials and Exponential Functions 172
Applied Project • Building a Better Roller Coaster 182
3.2 The Product and Quotient Rules 183
3.3 Derivatives of Trigonometric Functions 190
3.4 The Chain Rule 197
Applied Project • Where Should a Pilot Start Descent? 208
3.5 Implicit Differentiation 208
Laboratory Project • Families of Implicit Curves 217
3.6 Derivatives of Logarithmic Functions 218
3.7 Rates of Change in the Natural and Social Sciences 224
3.8 Exponential Growth and Decay 237
Applied Project • Controlling Red Blood Cell Loss During Surgery 244
3.9 Related Rates 245
3.10 Linear Approximations and Differentials 251
Laboratory Project • Taylor Polynomials 258
3.11 Hyperbolic Functions 259
Review 266
Problems Plus 270

4 Applications of Differentiation 275

4.1 Maximum and Minimum Values 276
Applied Project • The Calculus of Rainbows 285
4.2 The Mean Value Theorem 287
4.3 How Derivatives Affect the Shape of a Graph 293
4.4 Indeterminate Forms and l'Hospital's Rule 304
Writing Project • The Origins of l'Hospital's Rule 314
4.5 Summary of Curve Sketching 315
4.6 Graphing with Calculus *and* Calculators 323
4.7 Optimization Problems 330
Applied Project • The Shape of a Can 343
Applied Project • Planes and Birds: Minimizing Energy 344
4.8 Newton's Method 345
4.9 Antiderivatives 350
Review 358
Problems Plus 363

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

5 Integrals 365

5.1 Areas and Distances 366
5.2 The Definite Integral 378
Discovery Project • Area Functions 391
5.3 The Fundamental Theorem of Calculus 392
5.4 Indefinite Integrals and the Net Change Theorem 402
Writing Project • Newton, Leibniz, and the Invention of Calculus 411
5.5 The Substitution Rule 412
Review 421
Problems Plus 425

6 Applications of Integration 427

6.1 Areas Between Curves 428
Applied Project • The Gini Index 436
6.2 Volumes 438
6.3 Volumes by Cylindrical Shells 449
6.4 Work 455
6.5 Average Value of a Function 461
Applied Project • Calculus and Baseball 464
Applied Project • Where to Sit at the Movies 465
Review 466
Problems Plus 468

7 Techniques of Integration 471

7.1 Integration by Parts 472
7.2 Trigonometric Integrals 479
7.3 Trigonometric Substitution 486
7.4 Integration of Rational Functions by Partial Fractions 493
7.5 Strategy for Integration 503
7.6 Integration Using Tables and Computer Algebra Systems 508
Discovery Project • Patterns in Integrals 513
7.7 Approximate Integration 514
7.8 Improper Integrals 527
Review 537
Problems Plus 540

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

8 Further Applications of Integration 543

8.1 Arc Length 544
Discovery Project • Arc Length Contest 550
8.2 Area of a Surface of Revolution 551
Discovery Project • Rotating on a Slant 557
8.3 Applications to Physics and Engineering 558
Discovery Project • Complementary Coffee Cups 568
8.4 Applications to Economics and Biology 569
8.5 Probability 573
Review 581
Problems Plus 583

9 Differential Equations 585

9.1 Modeling with Differential Equations 586
9.2 Direction Fields and Euler's Method 591
9.3 Separable Equations 599
Applied Project • How Fast Does a Tank Drain? 608
Applied Project • Which Is Faster, Going Up or Coming Down? 609
9.4 Models for Population Growth 610
9.5 Linear Equations 620
9.6 Predator-Prey Systems 627
Review 634
Problems Plus 637

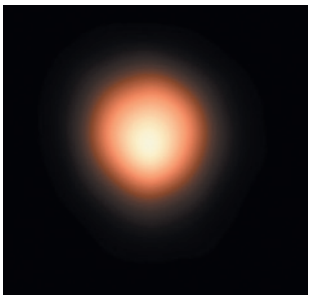
10 Parametric Equations and Polar Coordinates 639

10.1 Curves Defined by Parametric Equations 640
Laboratory Project • Running Circles Around Circles 648
10.2 Calculus with Parametric Curves 649
Laboratory Project • Bézier Curves 657
10.3 Polar Coordinates 658
Laboratory Project • Families of Polar Curves 668
10.4 Areas and Lengths in Polar Coordinates 669

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

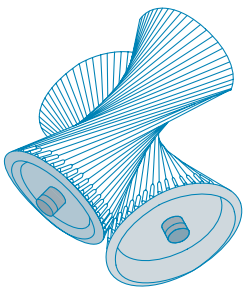
10.5 Conic Sections 674
10.6 Conic Sections in Polar Coordinates 682
Review 689
Problems Plus 692

11 Infinite Sequences and Series 693



11.1 Sequences 694
Laboratory Project • Logistic Sequences 707
11.2 Series 707
11.3 The Integral Test and Estimates of Sums 719
11.4 The Comparison Tests 727
11.5 Alternating Series 732
11.6 Absolute Convergence and the Ratio and Root Tests 737
11.7 Strategy for Testing Series 744
11.8 Power Series 746
11.9 Representations of Functions as Power Series 752
11.10 Taylor and Maclaurin Series 759
Laboratory Project • An Elusive Limit 773
Writing Project • How Newton Discovered the Binomial Series 773
11.11 Applications of Taylor Polynomials 774
Applied Project • Radiation from the Stars 783
Review 784
Problems Plus 787

12 Vectors and the Geometry of Space 791



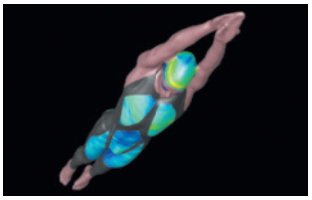
12.1 Three-Dimensional Coordinate Systems 792
12.2 Vectors 798
12.3 The Dot Product 807
12.4 The Cross Product 814
Discovery Project • The Geometry of a Tetrahedron 823
12.5 Equations of Lines and Planes 823
Laboratory Project • Putting 3D in Perspective 833
12.6 Cylinders and Quadric Surfaces 834
Review 841
Problems Plus 844

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

13 Vector Functions 847

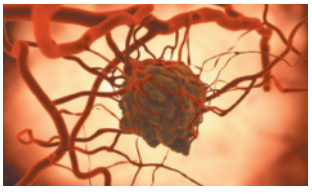
13.1 Vector Functions and Space Curves 848
13.2 Derivatives and Integrals of Vector Functions 855
13.3 Arc Length and Curvature 861
13.4 Motion in Space: Velocity and Acceleration 870
Applied Project • Kepler's Laws 880
Review 881
Problems Plus 884

14 Partial Derivatives 887



14.1 Functions of Several Variables 888
14.2 Limits and Continuity 903
14.3 Partial Derivatives 911
14.4 Tangent Planes and Linear Approximations 927
Applied Project • The Speedo LZR Racer 936
14.5 The Chain Rule 937
14.6 Directional Derivatives and the Gradient Vector 946
14.7 Maximum and Minimum Values 959
Applied Project • Designing a Dumpster 970
Discovery Project • Quadratic Approximations and Critical Points 970
14.8 Lagrange Multipliers 971
Applied Project • Rocket Science 979
Applied Project • Hydro-Turbine Optimization 980
Review 981
Problems Plus 985

15 Multiple Integrals 987



15.1 Double Integrals over Rectangles 988
15.2 Double Integrals over General Regions 1001
15.3 Double Integrals in Polar Coordinates 1010
15.4 Applications of Double Integrals 1016
15.5 Surface Area 1026

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

15.6 Triple Integrals 1029
Discovery Project • Volumes of Hyperspheres 1040
15.7 Triple Integrals in Cylindrical Coordinates 1040
Discovery Project • The Intersection of Three Cylinders 1044
15.8 Triple Integrals in Spherical Coordinates 1045
Applied Project • Roller Derby 1052
15.9 Change of Variables in Multiple Integrals 1052
Review 1061
Problems Plus 1065

16 Vector Calculus 1067

16.1 Vector Fields 1068
16.2 Line Integrals 1075
16.3 The Fundamental Theorem for Line Integrals 1087
16.4 Green's Theorem 1096
16.5 Curl and Divergence 1103
16.6 Parametric Surfaces and Their Areas 1111
16.7 Surface Integrals 1122
16.8 Stokes' Theorem 1134
Writing Project • Three Men and Two Theorems 1140
16.9 The Divergence Theorem 1141
16.10 Summary 1147
Review 1148
Problems Plus 1151

17 Second-Order Differential Equations 1153

17.1 Second-Order Linear Equations 1154
17.2 Nonhomogeneous Linear Equations 1160
17.3 Applications of Second-Order Differential Equations 1168
17.4 Series Solutions 1176
Review 1181

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Appendixes A1

A Numbers, Inequalities, and Absolute Values A2
B Coordinate Geometry and Lines A10
C Graphs of Second-Degree Equations A16
D Trigonometry A24
E Sigma Notation A34
F Proofs of Theorems A39
G The Logarithm Defined as an Integral A50
H Complex Numbers A57
I Answers to Odd-Numbered Exercises A65

Index A139

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Preface

This *International Metric Version* differs from the regular version of *Calculus: Early Transcendentals,* Eighth Edition, in several ways:

The units used in almost all of the examples and exercises have been changed from US Customary units to metric units. There are a small number of exceptions: In some engineering applications (principally in Section 8.3) it may be useful for some engineers to be familiar with US units. And I wanted to retain a few exercises (for example, those involving baseball) where it would be inappropriate to use metric units.

I've changed the examples and exercises involving real-world data to be more international in nature, so that the vast majority of them now come from countries other than the United States. For example, there are now exercises and examples concerning Hong Kong postal rates; Canadian public debt; unemployment rates in Australia; hours of daylight in Ankara, Turkey; isothermals in China; percentage of the population in rural Argentina; populations of Malaysia, Indonesia, Mexico, and India; and power consumption in Ontario, among many others.

In addition to changing exercises so that the units are metric and the data have a more international flavor, a number of other exercises have been changed as well, the result being that about 10% of the exercises are different from those in the regular version.

Philosophy of the Book

The art of teaching, Mark Van Doren said, is the art of assisting discovery. I have tried to write a book that assists students in discovering calculus—both for its practical power and its surprising beauty. In this edition, as in the first seven editions, I aim to convey to the student a sense of the utility of calculus and develop technical competence, but I also strive to give some appreciation for the intrinsic beauty of the subject. Newton undoubtedly experienced a sense of triumph when he made his great discoveries. I want students to share some of that excitement.

The emphasis is on understanding concepts. I think that nearly everybody agrees that this should be the primary goal of calculus instruction. In fact, the impetus for the current calculus reform movement came from the Tulane Conference in 1986, which formulated as their first recommendation:

Focus on conceptual understanding.

I have tried to implement this goal through the *Rule of Three:* "Topics should be presented geometrically, numerically, and algebraically." Visualization, numerical and graphical experimentation, and other approaches have changed how we teach conceptual reasoning in fundamental ways. More recently, the Rule of Three has been expanded to become the *Rule of Four* by emphasizing the verbal, or descriptive, point of view as well.

In writing the eighth edition my premise has been that it is possible to achieve conceptual understanding and still retain the best traditions of traditional calculus. The book contains elements of reform, but within the context of a traditional curriculum.

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Alternate Versions

I have written several other calculus textbooks that might be preferable for some instructors. Most of them also come in single variable and multivariable versions.

- *Calculus,* Eighth Edition, International Metric Version, is similar to the present textbook except that the exponential, logarithmic, and inverse trigonometric functions are covered in the second semester.
- *Essential Calculus*, Second Edition, International Edition, is a much briefer book (840 pages), though it contains almost all of the topics in *Calculus,* Eighth Edition, International Metric Version. The relative brevity is achieved through briefer exposition of some topics and putting some features on the website.
- *Essential Calculus: Early Transcendentals,* Second Edition, International Edition, resembles *Essential Calculus,* International Edition, but the exponential, logarithmic, and inverse trigonometric functions are covered in Chapter 3.
- *Calculus: Concepts and Contexts*, Fourth Edition, Metric International Edition, emphasizes conceptual understanding even more strongly than this book. The coverage of topics is not encyclopedic and the material on transcendental functions and on parametric equations is woven throughout the book instead of being treated in separate chapters.
- *Calculus: Early Vectors* introduces vectors and vector functions in the first semester and integrates them throughout the book. It is suitable for students taking engineering and physics courses concurrently with calculus.
- *Brief Applied Calculus,* International Edition, is intended for students in business, the social sciences, and the life sciences.
- *Biocalculus: Calculus for the Life Sciences* is intended to show students in the life sciences how calculus relates to biology.
- *Biocalculus: Calculus, Probability, and Statistics for the Life Sciences* contains all the content of *Biocalculus: Calculus for the Life Sciences* as well as three additional chapters covering probability and statistics.

What's New in the Eighth Edition?

The changes have resulted from talking with my colleagues and students at the University of Toronto and from reading journals, as well as suggestions from users and reviewers. Here are some of the many improvements that I've incorporated into this edition:

- The data in examples and exercises have been updated to be more timely.
- New examples have been added (see Examples 6.1.5, 11.2.5, and 14.3.3, for instance). And the solutions to some of the existing examples have been amplified.
- Three new projects have been added: The project *Controlling Red Blood Cell Loss During Surgery* (page 244) describes the ANH procedure, in which blood is extracted from the patient before an operation and is replaced by saline solution. This dilutes the patient's blood so that fewer red blood cells are lost during bleeding and the extracted blood is returned to the patient after surgery. The project *Planes and Birds: Minimizing Energy* (page 344) asks how birds can minimize power and energy by flapping their wings versus gliding. In the project *The Speedo LZR Racer* (page 936) it is explained that this suit reduces drag in the water and, as

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

a result, many swimming records were broken. Students are asked why a small decrease in drag can have a big effect on performance.

- I have streamlined Chapter 15 (Multiple Integrals) by combining the first two sections so that iterated integrals are treated earlier.
- More than 20% of the exercises in each chapter are new. Here are some of my favorites: 2.7.61, 2.8.36–38, 3.1.79–80, 3.11.54, 4.1.69, 4.3.34, 4.3.66, 4.4.80, 4.7.39, 4.7.67, 5.1.19–20, 5.2.67–68, 5.4.70, 6.1.51, 8.1.39, 12.5.81, 12.6.29–30, 14.6.65–66. In addition, there are some good new Problems Plus. (See Problems 12–14 on page 272, Problem 13 on page 363, Problems 16–17 on page 426, and Problem 8 on page 986.)

Features

Conceptual Exercises

The most important way to foster conceptual understanding is through the problems that we assign. To that end I have devised various types of problems. Some exercise sets begin with requests to explain the meanings of the basic concepts of the section. (See, for instance, the first few exercises in Sections 2.2, 2.5, 11.2, 14.2, and 14.3.) Similarly, all the review sections begin with a Concept Check and a True-False Quiz. Other exercises test conceptual understanding through graphs or tables (see Exercises 2.7.17, 2.8.35–38, 2.8.47–52, 9.1.11–13, 10.1.24–27, 11.10.2, 13.2.1–2, 13.3.33–39, 14.1.1–2, 14.1.32–38, 14.1.41–44, 14.3.3–10, 14.6.1–2, 14.7.3–4, 15.1.6–8, 16.1.11–18, 16.2.17–18, and 16.3.1–2).

Another type of exercise uses verbal description to test conceptual understanding (see Exercises 2.5.10, 2.8.66, 4.3.69–70, and 7.8.67). I particularly value problems that combine and compare graphical, numerical, and algebraic approaches (see Exercises 2.6.45–46, 3.7.27, and 9.4.4).

Graded Exercise Sets

Each exercise set is carefully graded, progressing from basic conceptual exercises and skill-development problems to more challenging problems involving applications and proofs.

Real-World Data

My assistants and I spent a great deal of time looking in libraries, contacting companies and government agencies, and searching the Internet for interesting real-world data to introduce, motivate, and illustrate the concepts of calculus. As a result, many of the examples and exercises deal with functions defined by such numerical data or graphs. See, for instance, Figure 1 in Section 1.1 (seismograms from the Northridge earthquake), Exercise 2.8.35 (unemployment rates), Exercise 5.1.16 (velocity of the space shuttle *Endeavour*), and Figure 4 in Section 5.4 (San Francisco power consumption). Functions of two variables are illustrated by a table of values of the wind-chill index as a function of air temperature and wind speed (Example 14.1.2). Partial derivatives are introduced in Section 14.3 by examining a column in a table of values of the heat index (perceived air temperature) as a function of the actual temperature and the relative humidity. This example is pursued further in connection with linear approximations (Example 14.4.3). Directional derivatives are introduced in Section 14.6 by using a temperature contour map to estimate the rate of change of temperature at Reno in the direction of Las Vegas. Double integrals are used to estimate the average snowfall in Colorado on December

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

20–21, 2006 (Example 15.1.9). Vector fields are introduced in Section 16.1 by depictions of actual velocity vector fields showing San Francisco Bay wind patterns.

Projects

One way of involving students and making them active learners is to have them work (perhaps in groups) on extended projects that give a feeling of substantial accomplishment when completed. I have included four kinds of projects: *Applied Projects* involve applications that are designed to appeal to the imagination of students. The project after Section 9.3 asks whether a ball thrown upward takes longer to reach its maximum height or to fall back to its original height. (The answer might surprise you.) The project after Section 14.8 uses Lagrange multipliers to determine the masses of the three stages of a rocket so as to minimize the total mass while enabling the rocket to reach a desired velocity. *Laboratory Projects* involve technology; the one following Section 10.2 shows how to use Bézier curves to design shapes that represent letters for a laser printer. *Writing Projects* ask students to compare present-day methods with those of the founders of calculus—Fermat's method for finding tangents, for instance. Suggested references are supplied. *Discovery Projects* anticipate results to be discussed later or encourage discovery through pattern recognition (see the one following Section 7.6). Others explore aspects of geometry: tetrahedra (after Section 12.4), hyperspheres (after Section 15.6), and intersections of three cylinders (after Section 15.7). Additional projects can be found in the *Instructor's Guide* (see, for instance, Group Exercise 5.1: Position from Samples).

Problem Solving

Students usually have difficulties with problems for which there is no single well-defined procedure for obtaining the answer. I think nobody has improved very much on George Polya's four-stage problem-solving strategy and, accordingly, I have included a version of his problem-solving principles following Chapter 1. They are applied, both explicitly and implicitly, throughout the book. After the other chapters I have placed sections called *Problems Plus*, which feature examples of how to tackle challenging calculus problems. In selecting the varied problems for these sections I kept in mind the following advice from David Hilbert: "A mathematical problem should be difficult in order to entice us, yet not inaccessible lest it mock our efforts." When I put these challenging problems on assignments and tests I grade them in a different way. Here I reward a student significantly for ideas toward a solution and for recognizing which problem-solving principles are relevant.

Technology

The availability of technology makes it not less important but more important to clearly understand the concepts that underlie the images on the screen. But, when properly used, graphing calculators and computers are powerful tools for discovering and understanding those concepts. This textbook can be used either with or without technology and I use two special symbols to indicate clearly when a particular type of machine is required. The icon indicates an exercise that definitely requires the use of such technology, but that is not to say that it can't be used on the other exercises as well. The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Maple, Mathematica, or the TI-89) are required. But technology doesn't make pencil and paper obsolete. Hand calculation and sketches are often preferable to technology for illustrating and reinforcing some concepts. Both instructors and students need to develop the ability to decide where the hand or the machine is appropriate.

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Tools for Enriching Calculus

TEC is a companion to the text and is intended to enrich and complement its contents. (It is now accessible in the eBook via CourseMate and Enhanced WebAssign. Selected Visuals and Modules are available at www.stewartcalculus.com.) Developed by Harvey Keynes, Dan Clegg, Hubert Hohn, and myself, TEC uses a discovery and exploratory approach. In sections of the book where technology is particularly appropriate, marginal icons direct students to TEC Modules that provide a laboratory environment in which they can explore the topic in different ways and at different levels. **Visuals are animations of figures in text; Modules are more elaborate activities and include exercises**. Instructors can choose to become involved at several different levels, ranging from simply encouraging students to use the Visuals and Modules for independent exploration, to assigning specific exercises from those included with each Module, or to creating additional exercises, labs, and projects that make use of the Visuals and Modules.

TEC also includes Homework Hints for representative exercises (usually odd-numbered) in every section of the text, indicated by printing the exercise number in red. These hints are usually presented in the form of questions and try to imitate an effective teaching assistant by functioning as a silent tutor. They are constructed so as not to reveal any more of the actual solution than is minimally necessary to make further progress.

Enhanced WebAssign

Technology is having an impact on the way homework is assigned to students, particularly in large classes. The use of online homework is growing and its appeal depends on ease of use, grading precision, and reliability. With the Eighth Edition we have been working with the calculus community and WebAssign to develop an online homework system. Up to 70% of the exercises in each section are assignable as online homework, including free response, multiple choice, and multi-part formats.

The system also includes Active Examples, in which students are guided in step-by-step tutorials through text examples, with links to the textbook and to video solutions.

Website

Visit CengageBrain.com or stewartcalculus.com for these additional materials:

- Homework Hints
- Algebra Review
- Lies My Calculator and Computer Told Me
- History of Mathematics, with links to the better historical websites
- Additional Topics (complete with exercise sets): Fourier Series, Formulas for the Remainder Term in Taylor Series, Rotation of Axes
- Archived Problems (Drill exercises that appeared in previous editions, together with their solutions)
- Challenge Problems (some from the Problems Plus sections from prior editions)
- Links, for particular topics, to outside Web resources
- Selected Visuals and Modules from Tools for Enriching Calculus (TEC)

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Content

Diagnostic Tests The book begins with four diagnostic tests, in Basic Algebra, Analytic Geometry, Functions, and Trigonometry.

A Preview of Calculus This is an overview of the subject and includes a list of questions to motivate the study of calculus.

1 Functions and Models From the beginning, multiple representations of functions are stressed: verbal, numerical, visual, and algebraic. A discussion of mathematical models leads to a review of the standard functions, including exponential and logarithmic functions, from these four points of view.

2 Limits and Derivatives The material on limits is motivated by a prior discussion of the tangent and velocity problems. Limits are treated from descriptive, graphical, numerical, and algebraic points of view. Section 2.4, on the precise definition of a limit, is an optional section. Sections 2.7 and 2.8 deal with derivatives (especially with functions defined graphically and numerically) before the differentiation rules are covered in Chapter 3. Here the examples and exercises explore the meanings of derivatives in various contexts. Higher derivatives are introduced in Section 2.8.

3 Differentiation Rules All the basic functions, including exponential, logarithmic, and inverse trigonometric functions, are differentiated here. When derivatives are computed in applied situations, students are asked to explain their meanings. Exponential growth and decay are now covered in this chapter.

4 Applications of Differentiation The basic facts concerning extreme values and shapes of curves are deduced from the Mean Value Theorem. Graphing with technology emphasizes the interaction between calculus and calculators and the analysis of families of curves. Some substantial optimization problems are provided, including an explanation of why you need to raise your head 42° to see the top of a rainbow.

5 Integrals The area problem and the distance problem serve to motivate the definite integral, with sigma notation introduced as needed. (Full coverage of sigma notation is provided in Appendix E.) Emphasis is placed on explaining the meanings of integrals in various contexts and on estimating their values from graphs and tables.

6 Applications of Integration Here I present the applications of integration—area, volume, work, average value—that can reasonably be done without specialized techniques of integration. General methods are emphasized. The goal is for students to be able to divide a quantity into small pieces, estimate with Riemann sums, and recognize the limit as an integral.

7 Techniques of Integration All the standard methods are covered but, of course, the real challenge is to be able to recognize which technique is best used in a given situation. Accordingly, in Section 7.5, I present a strategy for integration. The use of computer algebra systems is discussed in Section 7.6.

8 Further Applications of Integration Here are the applications of integration—arc length and surface area—for which it is useful to have available all the techniques of integration, as well as applications to biology, economics, and physics (hydrostatic force and centers of mass). I have also included a section on probability. There are more applications here than can realistically be covered in a given course. Instructors should select applications suitable for their students and for which they themselves have enthusiasm.

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

9 Differential Equations

Modeling is the theme that unifies this introductory treatment of differential equations. Direction fields and Euler's method are studied before separable and linear equations are solved explicitly, so that qualitative, numerical, and analytic approaches are given equal consideration. These methods are applied to the exponential, logistic, and other models for population growth. The first four or five sections of this chapter serve as a good introduction to first-order differential equations. An optional final section uses predator-prey models to illustrate systems of differential equations.

10 Parametric Equations and Polar Coordinates

This chapter introduces parametric and polar curves and applies the methods of calculus to them. Parametric curves are well suited to laboratory projects; the two presented here involve families of curves and Bézier curves. A brief treatment of conic sections in polar coordinates prepares the way for Kepler's Laws in Chapter 13.

11 Infinite Sequences and Series

The convergence tests have intuitive justifications (see page 719) as well as formal proofs. Numerical estimates of sums of series are based on which test was used to prove convergence. The emphasis is on Taylor series and polynomials and their applications to physics. Error estimates include those from graphing devices.

12 Vectors and the Geometry of Space

The material on three-dimensional analytic geometry and vectors is divided into two chapters. Chapter 12 deals with vectors, the dot and cross products, lines, planes, and surfaces.

13 Vector Functions

This chapter covers vector-valued functions, their derivatives and integrals, the length and curvature of space curves, and velocity and acceleration along space curves, culminating in Kepler's laws.

14 Partial Derivatives

Functions of two or more variables are studied from verbal, numerical, visual, and algebraic points of view. In particular, I introduce partial derivatives by looking at a specific column in a table of values of the heat index (perceived air temperature) as a function of the actual temperature and the relative humidity.

15 Multiple Integrals

Contour maps and the Midpoint Rule are used to estimate the average snowfall and average temperature in given regions. Double and triple integrals are used to compute probabilities, surface areas, and (in projects) volumes of hyperspheres and volumes of intersections of three cylinders. Cylindrical and spherical coordinates are introduced in the context of evaluating triple integrals.

16 Vector Calculus

Vector fields are introduced through pictures of velocity fields showing San Francisco Bay wind patterns. The similarities among the Fundamental Theorem for line integrals, Green's Theorem, Stokes' Theorem, and the Divergence Theorem are emphasized.

17 Second-Order Differential Equations

Since first-order differential equations are covered in Chapter 9, this final chapter deals with second-order linear differential equations, their application to vibrating springs and electric circuits, and series solutions.

Ancillaries

Calculus, Early Transcendentals, Eighth Edition, International Metric Version, is supported by a complete set of ancillaries developed under my direction. Each piece has been designed to enhance student understanding and to facilitate creative instruction. The tables on pages xxi–xxii describe each of these ancillaries.

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Acknowledgments

The preparation of this and previous editions has involved much time spent reading the reasoned (but sometimes contradictory) advice from a large number of astute reviewers. I greatly appreciate the time they spent to understand my motivation for the approach taken. I have learned something from each of them.

Eighth Edition Reviewers

Jay Abramson, *Arizona State University*
Adam Bowers, *University of California San Diego*
Neena Chopra, *The Pennsylvania State University*
Edward Dobson, *Mississippi State University*
Isaac Goldbring, *University of Illinois at Chicago*
Lea Jenkins, *Clemson University*
Rebecca Wahl, *Butler University*

Technology Reviewers

Maria Andersen, *Muskegon Community College*
Eric Aurand, *Eastfield College*
Joy Becker, *University of Wisconsin–Stout*
Przemyslaw Bogacki, *Old Dominion University*
Amy Elizabeth Bowman, *University of Alabama in Huntsville*
Monica Brown, *University of Missouri–St. Louis*
Roxanne Byrne, *University of Colorado at Denver and Health Sciences Center*
Teri Christiansen, *University of Missouri–Columbia*
Bobby Dale Daniel, *Lamar University*
Jennifer Daniel, *Lamar University*
Andras Domokos, *California State University, Sacramento*
Timothy Flaherty, *Carnegie Mellon University*
Lee Gibson, *University of Louisville*
Jane Golden, *Hillsborough Community College*
Semion Gutman, *University of Oklahoma*
Diane Hoffoss, *University of San Diego*
Lorraine Hughes, *Mississippi State University*
Jay Jahangiri, *Kent State University*
John Jernigan, *Community College of Philadelphia*
Brian Karasek, *South Mountain Community College*
Jason Kozinski, *University of Florida*
Carole Krueger, *The University of Texas at Arlington*
Ken Kubota, *University of Kentucky*
John Mitchell, *Clark College*
Donald Paul, *Tulsa Community College*
Chad Pierson, *University of Minnesota, Duluth*
Lanita Presson, *University of Alabama in Huntsville*
Karin Reinhold, *State University of New York at Albany*
Thomas Riedel, *University of Louisville*
Christopher Schroeder, *Morehead State University*
Angela Sharp, *University of Minnesota, Duluth*
Patricia Shaw, *Mississippi State University*
Carl Spitznagel, *John Carroll University*
Mohammad Tabanjeh, *Virginia State University*
Capt. Koichi Takagi, *United States Naval Academy*
Lorna TenEyck, *Chemeketa Community College*
Roger Werbylo, *Pima Community College*
David Williams, *Clayton State University*
Zhuan Ye, *Northern Illinois University*

Previous Edition Reviewers

B. D. Aggarwala, *University of Calgary*
John Alberghini, *Manchester Community College*
Michael Albert, *Carnegie-Mellon University*
Daniel Anderson, *University of Iowa*
Amy Austin, *Texas A&M University*
Donna J. Bailey, *Northeast Missouri State University*
Wayne Barber, *Chemeketa Community College*
Marilyn Belkin, *Villanova University*
Neil Berger, *University of Illinois, Chicago*
David Berman, *University of New Orleans*
Anthony J. Bevelacqua, *University of North Dakota*
Richard Biggs, *University of Western Ontario*
Robert Blumenthal, *Oglethorpe University*
Martina Bode, *Northwestern University*
Barbara Bohannon, *Hofstra University*
Jay Bourland, *Colorado State University*
Philip L. Bowers, *Florida State University*
Amy Elizabeth Bowman, *University of Alabama in Huntsville*
Stephen W. Brady, *Wichita State University*
Michael Breen, *Tennessee Technological University*
Robert N. Bryan, *University of Western Ontario*
David Buchthal, *University of Akron*
Jenna Carpenter, *Louisiana Tech University*
Jorge Cassio, *Miami-Dade Community College*
Jack Ceder, *University of California, Santa Barbara*
Scott Chapman, *Trinity University*

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Zhen-Qing Chen, *University of Washington—Seattle*
James Choike, *Oklahoma State University*
Barbara Cortzen, *DePaul University*
Carl Cowen, *Purdue University*
Philip S. Crooke, *Vanderbilt University*
Charles N. Curtis, *Missouri Southern State College*
Daniel Cyphert, *Armstrong State College*
Robert Dahlin
M. Hilary Davies, *University of Alaska Anchorage*
Gregory J. Davis, *University of Wisconsin–Green Bay*
Elias Deeba, *University of Houston–Downtown*
Daniel DiMaria, *Suffolk Community College*
Seymour Ditor, *University of Western Ontario*
Greg Dresden, *Washington and Lee University*
Daniel Drucker, *Wayne State University*
Kenn Dunn, *Dalhousie University*
Dennis Dunninger, *Michigan State University*
Bruce Edwards, *University of Florida*
David Ellis, *San Francisco State University*
John Ellison, *Grove City College*
Martin Erickson, *Truman State University*
Garret Etgen, *University of Houston*
Theodore G. Faticoni, *Fordham University*
Laurene V. Fausett, *Georgia Southern University*
Norman Feldman, *Sonoma State University*
Le Baron O. Ferguson, *University of California—Riverside*
Newman Fisher, *San Francisco State University*
José D. Flores, *The University of South Dakota*
William Francis, *Michigan Technological University*
James T. Franklin, *Valencia Community College, East*
Stanley Friedlander, *Bronx Community College*
Patrick Gallagher, *Columbia University–New York*
Paul Garrett, *University of Minnesota–Minneapolis*
Frederick Gass, *Miami University of Ohio*
Bruce Gilligan, *University of Regina*
Matthias K. Gobbert, *University of Maryland, Baltimore County*
Gerald Goff, *Oklahoma State University*
Stuart Goldenberg, *California Polytechnic State University*
John A. Graham, *Buckingham Browne & Nichols School*
Richard Grassl, *University of New Mexico*
Michael Gregory, *University of North Dakota*
Charles Groetsch, *University of Cincinnati*
Paul Triantafilos Hadavas, *Armstrong Atlantic State University*
Salim M. Haïdar, *Grand Valley State University*
D. W. Hall, *Michigan State University*
Robert L. Hall, *University of Wisconsin–Milwaukee*
Howard B. Hamilton, *California State University, Sacramento*
Darel Hardy, *Colorado State University*
Shari Harris, *John Wood Community College*
Gary W. Harrison, *College of Charleston*
Melvin Hausner, *New York University/Courant Institute*
Curtis Herink, *Mercer University*
Russell Herman, *University of North Carolina at Wilmington*
Allen Hesse, *Rochester Community College*
Randall R. Holmes, *Auburn University*
James F. Hurley, *University of Connecticut*
Amer Iqbal, *University of Washington—Seattle*
Matthew A. Isom, *Arizona State University*
Gerald Janusz, *University of Illinois at Urbana-Champaign*
John H. Jenkins, *Embry-Riddle Aeronautical University, Prescott Campus*
Clement Jeske, *University of Wisconsin, Platteville*
Carl Jockusch, *University of Illinois at Urbana-Champaign*
Jan E. H. Johansson, *University of Vermont*
Jerry Johnson, *Oklahoma State University*
Zsuzsanna M. Kadas, *St. Michael's College*
Nets Katz, *Indiana University Bloomington*
Matt Kaufman
Matthias Kawski, *Arizona State University*
Frederick W. Keene, *Pasadena City College*
Robert L. Kelley, *University of Miami*
Akhtar Khan, *Rochester Institute of Technology*
Marianne Korten, *Kansas State University*
Virgil Kowalik, *Texas A&I University*
Kevin Kreider, *University of Akron*
Leonard Krop, *DePaul University*
Mark Krusemeyer, *Carleton College*
John C. Lawlor, *University of Vermont*
Christopher C. Leary, *State University of New York at Geneseo*
David Leeming, *University of Victoria*
Sam Lesseig, *Northeast Missouri State University*
Phil Locke, *University of Maine*
Joyce Longman, *Villanova University*
Joan McCarter, *Arizona State University*
Phil McCartney, *Northern Kentucky University*
Igor Malyshev, *San Jose State University*
Larry Mansfield, *Queens College*
Mary Martin, *Colgate University*
Nathaniel F. G. Martin, *University of Virginia*
Gerald Y. Matsumoto, *American River College*
James McKinney, *California State Polytechnic University, Pomona*
Tom Metzger, *University of Pittsburgh*
Richard Millspaugh, *University of North Dakota*
Lon H. Mitchell, *Virginia Commonwealth University*
Michael Montaño, *Riverside Community College*
Teri Jo Murphy, *University of Oklahoma*
Martin Nakashima, *California State Polytechnic University, Pomona*
Ho Kuen Ng, *San Jose State University*
Richard Nowakowski, *Dalhousie University*
Hussain S. Nur, *California State University, Fresno*
Norma Ortiz-Robinson, *Virginia Commonwealth University*
Wayne N. Palmer, *Utica College*
Vincent Panico, *University of the Pacific*
F. J. Papp, *University of Michigan–Dearborn*
Mike Penna, *Indiana University–Purdue University Indianapolis*
Mark Pinsky, *Northwestern University*
Lothar Redlin, *The Pennsylvania State University*
Joel W. Robbin, *University of Wisconsin–Madison*
Lila Roberts, *Georgia College and State University*
E. Arthur Robinson, Jr., *The George Washington University*
Richard Rockwell, *Pacific Union College*

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Rob Root, *Lafayette College*
Richard Ruedemann, *Arizona State University*
David Ryeburn, *Simon Fraser University*
Richard St. Andre, *Central Michigan University*
Ricardo Salinas, *San Antonio College*
Robert Schmidt, *South Dakota State University*
Eric Schreiner, *Western Michigan University*
Mihr J. Shah, *Kent State University–Trumbull*
Qin Sheng, *Baylor University*
Theodore Shifrin, *University of Georgia*
Wayne Skrapek, *University of Saskatchewan*
Larry Small, *Los Angeles Pierce College*
Teresa Morgan Smith, *Blinn College*
William Smith, *University of North Carolina*
Donald W. Solomon, *University of Wisconsin–Milwaukee*
Edward Spitznagel, *Washington University*
Joseph Stampfli, *Indiana University*
Kristin Stoley, *Blinn College*
M. B. Tavakoli, *Chaffey College*
Magdalena Toda, *Texas Tech University*
Ruth Trygstad, *Salt Lake Community College*
Paul Xavier Uhlig, *St. Mary's University, San Antonio*
Stan Ver Nooy, *University of Oregon*
Andrei Verona, *California State University–Los Angeles*
Klaus Volpert, *Villanova University*
Russell C. Walker, *Carnegie Mellon University*
William L. Walton, *McCallie School*
Peiyong Wang, *Wayne State University*
Jack Weiner, *University of Guelph*
Alan Weinstein, *University of California, Berkeley*
Theodore W. Wilcox, *Rochester Institute of Technology*
Steven Willard, *University of Alberta*
Robert Wilson, *University of Wisconsin–Madison*
Jerome Wolbert, *University of Michigan–Ann Arbor*
Dennis H. Wortman, *University of Massachusetts, Boston*
Mary Wright, *Southern Illinois University–Carbondale*
Paul M. Wright, *Austin Community College*
Xian Wu, *University of South Carolina*

In addition, I would like to thank R. B. Burckel, Bruce Colletti, David Behrman, John Dersch, Gove Effinger, Bill Emerson, Dan Kalman, Quyan Khan, Alfonso Gracia-Saz, Allan MacIsaac, Tami Martin, Monica Nitsche, Lamia Raffo, Norton Starr, and Jim Trefzger for their suggestions; Al Shenk and Dennis Zill for permission to use exercises from their calculus texts; COMAP for permission to use project material; George Bergman, David Bleecker, Dan Clegg, Victor Kaftal, Anthony Lam, Jamie Lawson, Ira Rosenholtz, Paul Sally, Lowell Smylie, and Larry Wallen for ideas for exercises; Dan Drucker for the roller derby project; Thomas Banchoff, Tom Farmer, Fred Gass, John Ramsay, Larry Riddle, Philip Straffin, and Klaus Volpert for ideas for projects; Dan Anderson, Dan Clegg, Jeff Cole, Dan Drucker, and Barbara Frank for solving the new exercises and suggesting ways to improve them; Marv Riedesel and Mary Johnson for accuracy in proofreading; Andy Bulman-Fleming, Lothar Redlin, Gina Sanders, and Saleem Watson for additional proofreading; and Jeff Cole and Dan Clegg for their careful preparation and proofreading of the answer manuscript.

In addition, I thank those who have contributed to past editions: Ed Barbeau, George Bergman, Fred Brauer, Andy Bulman-Fleming, Bob Burton, David Cusick, Tom DiCiccio, Garret Etgen, Chris Fisher, Leon Gerber, Stuart Goldenberg, Arnold Good, Gene Hecht, Harvey Keynes, E. L. Koh, Zdislav Kovarik, Kevin Kreider, Emile LeBlanc, David Leep, Gerald Leibowitz, Larry Peterson, Mary Pugh, Lothar Redlin, Carl Riehm, John Ringland, Peter Rosenthal, Dusty Sabo, Doug Shaw, Dan Silver, Simon Smith, Saleem Watson, Alan Weinstein, and Gail Wolkowicz.

I also thank Kathi Townes, Stephanie Kuhns, Kristina Elliott, and Kira Abdallah of TECHarts for their production services and the following Cengage Learning staff: Cheryll Linthicum, content project manager; Stacy Green, senior content developer; Samantha Lugtu, associate content developer; Stephanie Kreuz, product assistant; Lynh Pham, media developer; Ryan Ahern, marketing manager; and Vernon Boes, art director. They have all done an outstanding job.

I have been very fortunate to have worked with some of the best mathematics editors in the business over the past three decades: Ron Munro, Harry Campbell, Craig Barth, Jeremy Hayhurst, Gary Ostedt, Bob Pirtle, Richard Stratton, Liz Covello, and now Neha Taleja. All of them have contributed greatly to the success of this book.

JAMES STEWART

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Ancillaries for Instructors

Instructor's Guide

by Douglas Shaw

ISBN 978-1-305-39371-4

Each section of the text is discussed from several viewpoints. The Instructor's Guide contains suggested time to allot, points to stress, text discussion topics, core materials for lecture, workshop/discussion suggestions, group work exercises in a form suitable for handout, and suggested homework assignments.

Complete Solutions Manual

Single Variable Early Transcendentals

ISBN 978-1-305-27262-0

Multivariable

ISBN 978-1-305-38699-0

Includes worked-out solutions to all exercises in the text.

Printed Test Bank

By William Steven Harmon

ISBN 978-1-305-38722-5

Contains text-specific multiple-choice and free response test items.

Cengage Learning Testing Powered by Cognero

(login.cengage.com)

This flexible online system allows you to author, edit, and manage test bank content from multiple Cengage Learning solutions; create multiple test versions in an instant; and deliver tests from your LMS, your classroom, or wherever you want.

Ancillaries for Instructors and Students

Stewart Website

www.stewartcalculus.com

Contents: *Homework Hints* ■ *Algebra Review* ■ *Additional Topics* ■ *Drill exercises* ■ *Challenge Problems* ■ *Web Links* ■ *History of Mathematics* ■ *Tools for Enriching Calculus (TEC)*

TEC TOOLS FOR ENRICHING™ CALCULUS

By James Stewart, Harvey Keynes, Dan Clegg, and developer Hubert Hohn

Tools for Enriching Calculus (TEC) functions as both a powerful tool for instructors and as a tutorial environment in which students can explore and review selected topics. The Flash simulation modules in TEC include instructions, written and audio explanations of the concepts, and exercises. TEC is accessible in the eBook via CourseMate and Enhanced WebAssign. Selected Visuals and Modules are available at www.stewartcalculus.com.

ENHANCED WebAssign Enhanced WebAssign®

www.webassign.net

Printed Access Code: ISBN 978-1-285-85826-5

Instant Access Code ISBN: 978-1-285-85825-8

Exclusively from Cengage Learning, Enhanced WebAssign offers an extensive online program for Stewart's Calculus *to encourage the practice that is so critical for concept mastery. The meticulously crafted pedagogy and exercises in our proven texts become even more effective in Enhanced WebAssign, supplemented by multimedia tutorial support and immediate feedback as students complete their assignments. Key features include:*

- *Thousands of homework problems that match your textbook's end-of-section exercises*
- *Opportunities for students to review prerequisite skills and content both at the start of the course and at the beginning of each section*
- Read It *eBook pages,* Watch It *videos,* Master It *tutorials, and* Chat About It *links*
- *A customizable Cengage* YouBook *with highlighting, note-taking, and search features, as well as links to multimedia resources*
- Personal Study Plans *(based on diagnostic quizzing) that identify chapter topics that students will need to master*
- *A WebAssign* Answer Evaluator *that recognizes and accepts equivalent mathematical responses in the same way an instructor grades*
- *A* Show My Work *feature that gives instructors the option of seeing students' detailed solutions*
- Visualizing Calculus Animations, Lecture Videos, *and more*

■ Electronic items ■ Printed items

(Table continues on page xxii)

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Cengage Customizable YouBook

YouBook *is an eBook that is both interactive and customizable. Containing all the content from Stewart's* Calculus, *YouBook features a text edit tool that allows instructors to modify the textbook narrative as needed. With* YouBook, *instructors can quickly reorder entire sections and chapters or hide any content they don't teach to create an eBook that perfectly matches their syllabus. Instructors can further customize the text by adding instructor-created or YouTube video links. Additional media assets include animated figures, video clips, highlighting and note-taking features, and more. YouBook is available within Enhanced WebAssign.*

CourseMate

CourseMate is a perfect self-study tool for students, and requires no set up from instructors. CourseMate brings course concepts to life with interactive learning, study, and exam preparation tools that support the printed textbook. CourseMate for Stewart's Calculus *includes an interactive eBook, Tools for Enriching Calculus, videos, quizzes, flashcards, and more. For instructors, CourseMate includes Engagement Tracker, a first-of-its-kind tool that monitors student engagement.*

CengageBrain.com

To access additional course materials, please visit www.cengagebrain.com. *At the CengageBrain.com home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where these resources can be found.*

Ancillaries for Students

Student Solutions Manual

Single Variable Early Transcendentals

ISBN 978-1-305-27263-7

Multivariable

ISBN 978-1-305-38698-3

Provides completely worked-out solutions to all odd-numbered exercises in the text, giving students a chance to check their answer and ensure they took the correct steps to arrive at the answer. The Student Solutions Manual can be ordered or accessed online as an eBook at www.cengagebrain.com *by searching the ISBN.*

Study Guide

Single Variable Early Transcendentals

By Richard St. Andre

ISBN 978-1-305-27914-8

Multivariable

By Richard St. Andre

ISBN 978-1-305-27184-5

For each section of the text, the Study Guide *provides students with a brief introduction, a short list of concepts to master, and summary and focus questions with explained answers. The* Study Guide *also contains self-tests with exam-style questions. The* Study Guide *can be ordered or accessed online as an eBook at* www.cengagebrain.com *by searching the ISBN.*

A Companion to Calculus

By Dennis Ebersole, Doris Schattschneider, Alicia Sevilla, and Kay Somers

ISBN 978-0-495-01124-8

Written to improve algebra and problem-solving skills of students taking a calculus course, every chapter in this companion is keyed to a calculus topic, providing conceptual background and specific algebra techniques needed to understand and solve calculus problems related to that topic. It is designed for calculus courses that integrate the review of precalculus concepts or for individual use. Order a copy of the text or access the eBook online at www.cengagebrain.com *by searching the ISBN.*

Linear Algebra for Calculus

by Konrad J. Heuvers, William P. Francis, John H. Kuisti, Deborah F. Lockhart, Daniel S. Moak, and Gene M. Ortner

ISBN 978-0-534-25248-9

This comprehensive book, designed to supplement the calculus course, provides an introduction to and review of the basic ideas of linear algebra. Order a copy of the text or access the eBook online at www.cengagebrain.com *by searching the ISBN.*

■ Electronic items ■ Printed items

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

To the Student

Reading a calculus textbook is different from reading a newspaper or a novel, or even a physics book. Don't be discouraged if you have to read a passage more than once in order to understand it. You should have pencil and paper and calculator at hand to sketch a diagram or make a calculation.

Some students start by trying their homework problems and read the text only if they get stuck on an exercise. I suggest that a far better plan is to read and understand a section of the text before attempting the exercises. In particular, you should look at the definitions to see the exact meanings of the terms. And before you read each example, I suggest that you cover up the solution and try solving the problem yourself. You'll get a lot more from looking at the solution if you do so.

Part of the aim of this course is to train you to think logically. Learn to write the solutions of the exercises in a connected, step-by-step fashion with explanatory sentences—not just a string of disconnected equations or formulas.

The answers to the odd-numbered exercises appear at the back of the book, in Appendix I. Some exercises ask for a verbal explanation or interpretation or description. In such cases there is no single correct way of expressing the answer, so don't worry that you haven't found the definitive answer. In addition, there are often several different forms in which to express a numerical or algebraic answer, so if your answer differs from mine, don't immediately assume you're wrong. For example, if the answer given in the back of the book is $\sqrt{2} - 1$ and you obtain $1/(1 + \sqrt{2})$, then you're right and rationalizing the denominator will show that the answers are equivalent.

The icon indicates an exercise that definitely requires the use of either a graphing calculator or a computer with graphing software. But that doesn't mean that graphing devices can't be used to check your work on the other exercises as well. The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Maple, Mathematica, or the TI-89) are required.

You will also encounter the symbol , which warns you against committing an error. I have placed this symbol in the margin in situations where I have observed that a large proportion of my students tend to make the same mistake.

Tools for Enriching Calculus, which is a companion to this text, is referred to by means of the symbol TEC and can be accessed in the eBook via Enhanced WebAssign and CourseMate (selected Visuals and Modules are available at www.stewartcalculus.com). It directs you to modules in which you can explore aspects of calculus for which the computer is particularly useful.

You will notice that some exercise numbers are printed in red: 5. This indicates that *Homework Hints* are available for the exercise. These hints can be found on stewartcalculus.com as well as Enhanced WebAssign and CourseMate. The homework hints ask you questions that allow you to make progress toward a solution without actually giving you the answer. You need to pursue each hint in an active manner with pencil and paper to work out the details. If a particular hint doesn't enable you to solve the problem, you can click to reveal the next hint.

I recommend that you keep this book for reference purposes after you finish the course. Because you will likely forget some of the specific details of calculus, the book will serve as a useful reminder when you need to use calculus in subsequent courses. And, because this book contains more material than can be covered in any one course, it can also serve as a valuable resource for a working scientist or engineer.

Calculus is an exciting subject, justly considered to be one of the greatest achievements of the human intellect. I hope you will discover that it is not only useful but also intrinsically beautiful.

JAMES STEWART

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

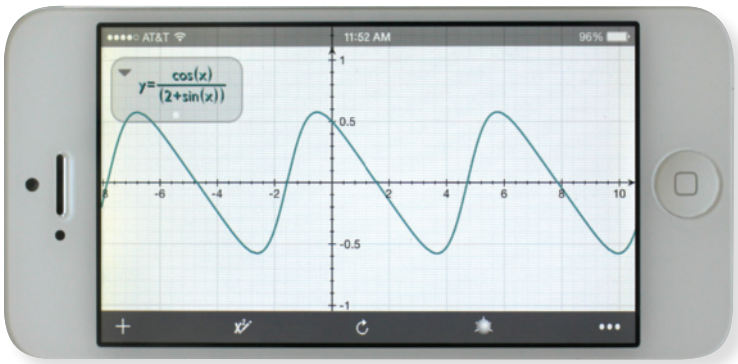
Calculators, Computers, and Other Graphing Devices

© Dan Clegg

Advances in technology continue to bring a wider variety of tools for doing mathematics. Handheld calculators are becoming more powerful, as are software programs and Internet resources. In addition, many mathematical applications have been released for smartphones and tablets such as the iPad.

Some exercises in this text are marked with a graphing icon, which indicates that the use of some technology is required. Often this means that we intend for a graphing device to be used in drawing the graph of a function or equation. You might also need technology to find the zeros of a graph or the points of intersection of two graphs. In some cases we will use a calculating device to solve an equation or evaluate a definite integral numerically. Many scientific and graphing calculators have these features built in, such as the Texas Instruments TI-84 or TI-Nspire CX. Similar calculators are made by Hewlett Packard, Casio, and Sharp.

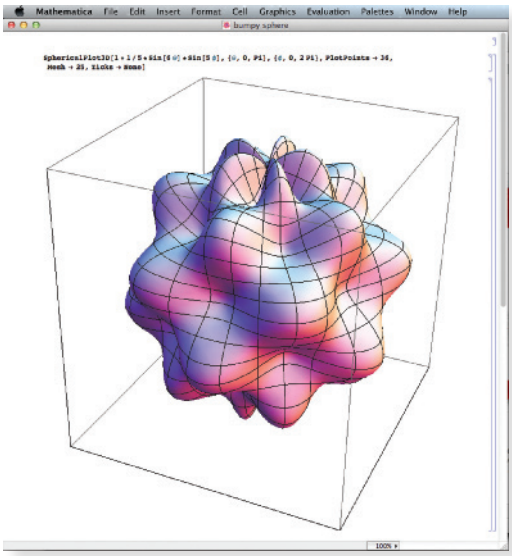
You can also use computer software such as *Graphing Calculator* by Pacific Tech (www.pacifict.com) to perform many of these functions, as well as apps for phones and tablets, like Quick Graph (Colombiamug) or Math-Studio (Pomegranate Apps). Similar functionality is available using a web interface at WolframAlpha.com.



© Dan Clegg

© Dan Clegg

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.



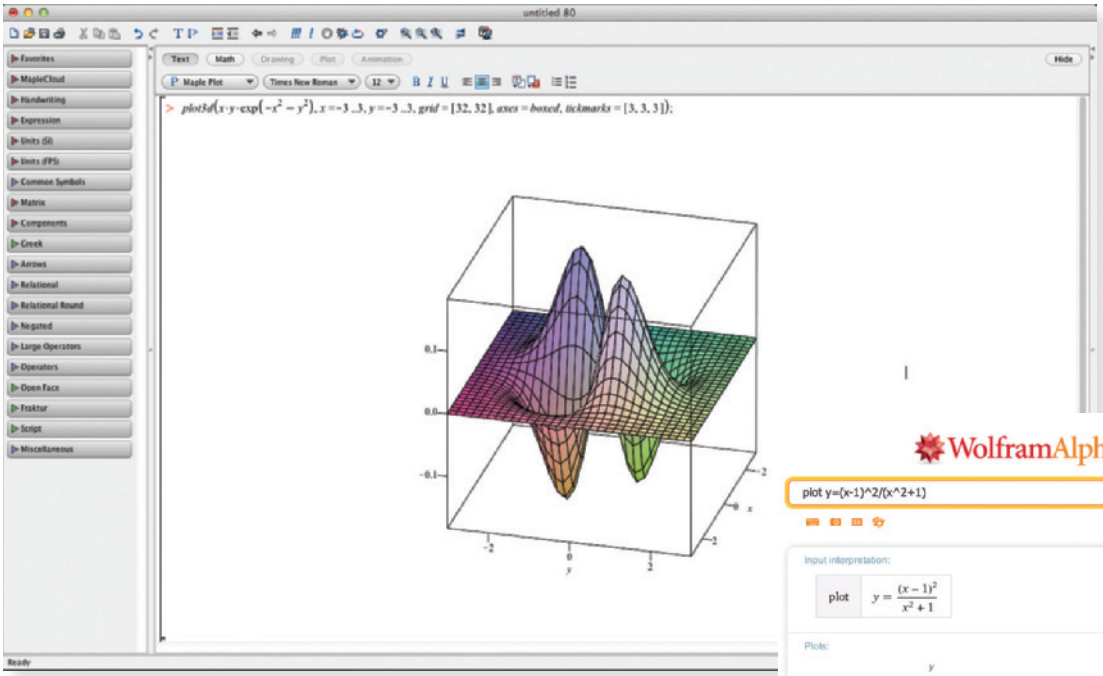
© Dan Clegg

In general, when we use the term "calculator" in this book, we mean the use of any of the resources we have mentioned.

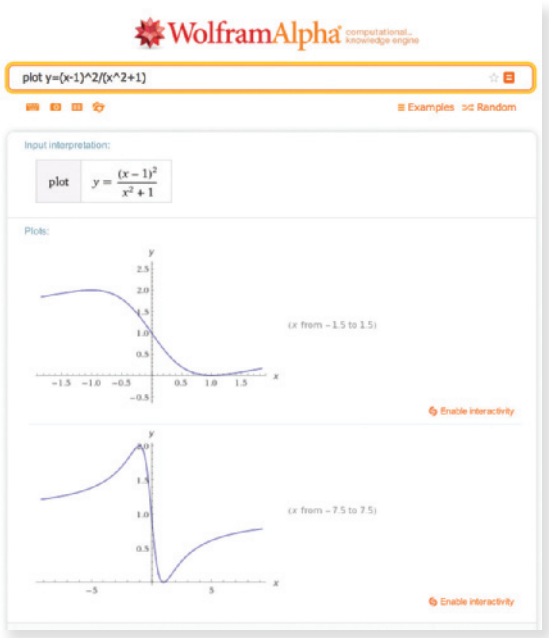
The CAS icon is reserved for problems in which the full resources of a *computer algebra system* (CAS) are required. A CAS is capable of doing mathematics (like solving equations, computing derivatives or integrals) *symbolically* rather than just numerically.

Examples of well-established computer algebra systems are the computer software packages Maple and Mathematica. The WolframAlpha website uses the Mathematica engine to provide CAS functionality via the Web.

Many handheld graphing calculators have CAS capabilities, such as the TI-89 and TI-Nspire CX CAS from Texas Instruments. Some tablet and smartphone apps also provide these capabilities, such as the previously mentioned MathStudio.



© Dan Clegg



© Dan Clegg

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Diagnostic Tests

Success in calculus depends to a large extent on knowledge of the mathematics that precedes calculus: algebra, analytic geometry, functions, and trigonometry. The following tests are intended to diagnose weaknesses that you might have in these areas. After taking each test you can check your answers against the given answers and, if necessary, refresh your skills by referring to the review materials that are provided.

A Diagnostic Test: Algebra

1. Evaluate each expression without using a calculator.

(a) $(-3)^4$ (b) -3^4 (c) 3^{-4}

(d) $\dfrac{5^{23}}{5^{21}}$ (e) $\left(\dfrac{2}{3}\right)^{-2}$ (f) $16^{-3/4}$

2. Simplify each expression. Write your answer without negative exponents.

(a) $\sqrt{200} - \sqrt{32}$

(b) $(3a^3b^3)(4ab^2)^2$

(c) $\left(\dfrac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

3. Expand and simplify.

(a) $3(x + 6) + 4(2x - 5)$ (b) $(x + 3)(4x - 5)$

(c) $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right)$ (d) $(2x + 3)^2$

(e) $(x + 2)^3$

4. Factor each expression.

(a) $4x^2 - 25$ (b) $2x^2 + 5x - 12$

(c) $x^3 - 3x^2 - 4x + 12$ (d) $x^4 + 27x$

(e) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$ (f) $x^3y - 4xy$

5. Simplify the rational expression.

(a) $\dfrac{x^2 + 3x + 2}{x^2 - x - 2}$ (b) $\dfrac{2x^2 - x - 1}{x^2 - 9} \cdot \dfrac{x + 3}{2x + 1}$

(c) $\dfrac{x^2}{x^2 - 4} - \dfrac{x + 1}{x + 2}$ (d) $\dfrac{\dfrac{y}{x} - \dfrac{x}{y}}{\dfrac{1}{y} - \dfrac{1}{x}}$

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

6. Rationalize the expression and simplify.

(a) $\dfrac{\sqrt{10}}{\sqrt{5}-2}$ (b) $\dfrac{\sqrt{4+h}-2}{h}$

7. Rewrite by completing the square.

(a) x^2+x+1 (b) $2x^2-12x+11$

8. Solve the equation. (Find only the real solutions.)

(a) $x+5=14-\frac{1}{2}x$ (b) $\dfrac{2x}{x+1}=\dfrac{2x-1}{x}$

(c) $x^2-x-12=0$ (d) $2x^2+4x+1=0$

(e) $x^4-3x^2+2=0$ (f) $3|x-4|=10$

(g) $2x(4-x)^{-1/2}-3\sqrt{4-x}=0$

9. Solve each inequality. Write your answer using interval notation.

(a) $-4<5-3x\leqslant 17$ (b) $x^2<2x+8$

(c) $x(x-1)(x+2)>0$ (d) $|x-4|<3$

(e) $\dfrac{2x-3}{x+1}\leqslant 1$

10. State whether each equation is true or false.

(a) $(p+q)^2=p^2+q^2$ (b) $\sqrt{ab}=\sqrt{a}\,\sqrt{b}$

(c) $\sqrt{a^2+b^2}=a+b$ (d) $\dfrac{1+TC}{C}=1+T$

(e) $\dfrac{1}{x-y}=\dfrac{1}{x}-\dfrac{1}{y}$ (f) $\dfrac{1/x}{a/x-b/x}=\dfrac{1}{a-b}$

ANSWERS TO DIAGNOSTIC TEST A: ALGEBRA

1. (a) 81 (b) -81 (c) $\frac{1}{81}$
(d) 25 (e) $\frac{9}{4}$ (f) $\frac{1}{8}$

2. (a) $6\sqrt{2}$ (b) $48a^5b^7$ (c) $\dfrac{x}{9y^7}$

3. (a) $11x-2$ (b) $4x^2+7x-15$
(c) $a-b$ (d) $4x^2+12x+9$
(e) $x^3+6x^2+12x+8$

4. (a) $(2x-5)(2x+5)$ (b) $(2x-3)(x+4)$
(c) $(x-3)(x-2)(x+2)$ (d) $x(x+3)(x^2-3x+9)$
(e) $3x^{-1/2}(x-1)(x-2)$ (f) $xy(x-2)(x+2)$

5. (a) $\dfrac{x+2}{x-2}$ (b) $\dfrac{x-1}{x-3}$
(c) $\dfrac{1}{x-2}$ (d) $-(x+y)$

6. (a) $5\sqrt{2}+2\sqrt{10}$ (b) $\dfrac{1}{\sqrt{4+h}+2}$

7. (a) $\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$ (b) $2(x-3)^2-7$

8. (a) 6 (b) 1 (c) $-3, 4$
(d) $-1\pm\frac{1}{2}\sqrt{2}$ (e) $\pm 1, \pm\sqrt{2}$ (f) $\frac{2}{3}, \frac{22}{3}$
(g) $\frac{12}{5}$

9. (a) $[-4, 3)$ (b) $(-2, 4)$
(c) $(-2, 0)\cup(1, \infty)$ (d) $(1, 7)$
(e) $(-1, 4]$

10. (a) False (b) True (c) False
(d) False (e) False (f) True

If you had difficulty with these problems, you may wish to consult the Review of Algebra on the website **www.stewartcalculus.com**.

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

B Diagnostic Test: Analytic Geometry

1. Find an equation for the line that passes through the point $(2, -5)$ and
(a) has slope -3
(b) is parallel to the x-axis
(c) is parallel to the y-axis
(d) is parallel to the line $2x - 4y = 3$

2. Find an equation for the circle that has center $(-1, 4)$ and passes through the point $(3, -2)$.

3. Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

4. Let $A(-7, 4)$ and $B(5, -12)$ be points in the plane.
(a) Find the slope of the line that contains A and B.
(b) Find an equation of the line that passes through A and B. What are the intercepts?
(c) Find the midpoint of the segment AB.
(d) Find the length of the segment AB.
(e) Find an equation of the perpendicular bisector of AB.
(f) Find an equation of the circle for which AB is a diameter.

5. Sketch the region in the xy-plane defined by the equation or inequalities.
(a) $-1 \leqslant y \leqslant 3$
(b) $|x| < 4$ and $|y| < 2$
(c) $y < 1 - \frac{1}{2}x$
(d) $y \geqslant x^2 - 1$
(e) $x^2 + y^2 < 4$
(f) $9x^2 + 16y^2 = 144$

ANSWERS TO DIAGNOSTIC TEST B: ANALYTIC GEOMETRY

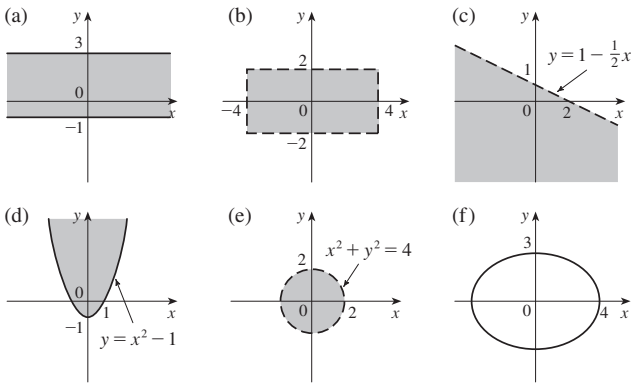
1. (a) $y = -3x + 1$
(b) $y = -5$
(c) $x = 2$
(d) $y = \frac{1}{2}x - 6$

2. $(x + 1)^2 + (y - 4)^2 = 52$

3. Center $(3, -5)$, radius 5

4. (a) $-\frac{4}{3}$
(b) $4x + 3y + 16 = 0$; x-intercept -4, y-intercept $-\frac{16}{3}$
(c) $(-1, -4)$
(d) 20
(e) $3x - 4y = 13$
(f) $(x + 1)^2 + (y + 4)^2 = 100$

5.



If you had difficulty with these problems, you may wish to consult the review of analytic geometry in Appendixes B and C.

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

C Diagnostic Test: Functions

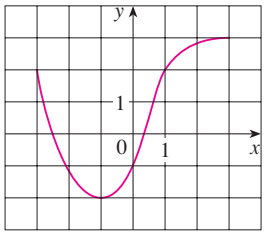


FIGURE FOR PROBLEM 1

1. The graph of a function f is given at the left.
(a) State the value of $f(-1)$.
(b) Estimate the value of $f(2)$.
(c) For what values of x is $f(x) = 2$?
(d) Estimate the values of x such that $f(x) = 0$.
(e) State the domain and range of f.

2. If $f(x) = x^3$, evaluate the difference quotient $\dfrac{f(2+h) - f(2)}{h}$ and simplify your answer.

3. Find the domain of the function.

(a) $f(x) = \dfrac{2x+1}{x^2+x-2}$ (b) $g(x) = \dfrac{\sqrt[3]{x}}{x^2+1}$ (c) $h(x) = \sqrt{4-x} + \sqrt{x^2-1}$

4. How are graphs of the functions obtained from the graph of f?

(a) $y = -f(x)$ (b) $y = 2f(x) - 1$ (c) $y = f(x-3) + 2$

5. Without using a calculator, make a rough sketch of the graph.

(a) $y = x^3$ (b) $y = (x+1)^3$ (c) $y = (x-2)^3 + 3$
(d) $y = 4 - x^2$ (e) $y = \sqrt{x}$ (f) $y = 2\sqrt{x}$
(g) $y = -2^x$ (h) $y = 1 + x^{-1}$

6. Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \le 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

(a) Evaluate $f(-2)$ and $f(1)$. (b) Sketch the graph of f.

7. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions.
(a) $f \circ g$ (b) $g \circ f$ (c) $g \circ g \circ g$

ANSWERS TO DIAGNOSTIC TEST C: FUNCTIONS

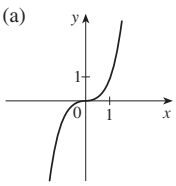
1. (a) -2 (b) 2.8
(c) $-3, 1$ (d) $-2.5, 0.3$
(e) $[-3, 3], [-2, 3]$

2. $12 + 6h + h^2$

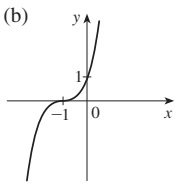
3. (a) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$
(b) $(-\infty, \infty)$
(c) $(-\infty, -1] \cup [1, 4]$

4. (a) Reflect about the x-axis
(b) Stretch vertically by a factor of 2, then shift 1 unit downward
(c) Shift 3 units to the right and 2 units upward

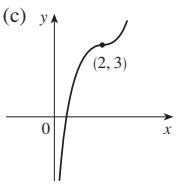
5. (a)



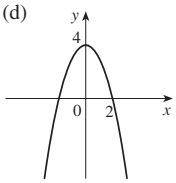
(b)



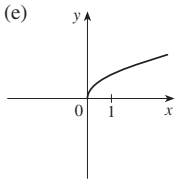
(c)



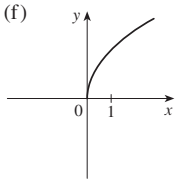
(d)



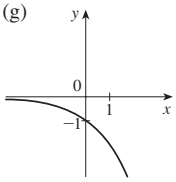
(e)



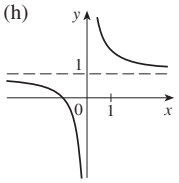
(f)



(g)

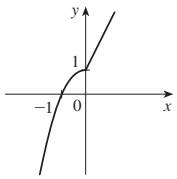


(h)



Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

6. (a) $-3, 3$ (b)



7. (a) $(f \circ g)(x) = 4x^2 - 8x + 2$
(b) $(g \circ f)(x) = 2x^2 + 4x - 5$
(c) $(g \circ g \circ g)(x) = 8x - 21$

If you had difficulty with these problems, you should look at sections 1.1–1.3 of this book.

D Diagnostic Test: Trigonometry

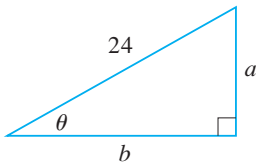


FIGURE FOR PROBLEM 5

1. Convert from degrees to radians.
(a) $300°$ (b) $-18°$

2. Convert from radians to degrees.
(a) $5\pi/6$ (b) 2

3. Find the length of an arc of a circle with radius 12 cm if the arc subtends a central angle of $30°$.

4. Find the exact values.
(a) $\tan(\pi/3)$ (b) $\sin(7\pi/6)$ (c) $\sec(5\pi/3)$

5. Express the lengths a and b in the figure in terms of θ.

6. If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where x and y lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.

7. Prove the identities.
(a) $\tan\theta \sin\theta + \cos\theta = \sec\theta$ (b) $\dfrac{2\tan x}{1 + \tan^2 x} = \sin 2x$

8. Find all values of x such that $\sin 2x = \sin x$ and $0 \leqslant x \leqslant 2\pi$.

9. Sketch the graph of the function $y = 1 + \sin 2x$ without using a calculator.

ANSWERS TO DIAGNOSTIC TEST D: TRIGONOMETRY

1. (a) $5\pi/3$ (b) $-\pi/10$

2. (a) $150°$ (b) $360°/\pi \approx 114.6°$

3. 2π cm

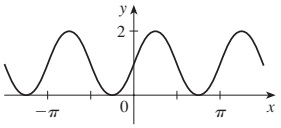
4. (a) $\sqrt{3}$ (b) $-\frac{1}{2}$ (c) 2

5. (a) $24\sin\theta$ (b) $24\cos\theta$

6. $\frac{1}{15}\left(4 + 6\sqrt{2}\right)$

8. $0, \pi/3, \pi, 5\pi/3, 2\pi$

9.



If you had difficulty with these problems, you should look at Appendix D of this book.

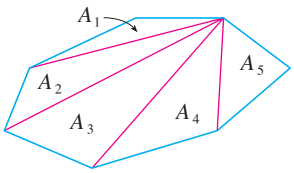
Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

A Preview of Calculus

By the time you finish this course, you will be able to calculate the length of the curve used to design the Gateway Arch in St. Louis, determine where a pilot should start descent for a smooth landing, compute the force on a baseball bat when it strikes the ball, and measure the amount of light sensed by the human eye as the pupil changes size.

CALCULUS IS FUNDAMENTALLY DIFFERENT FROM the mathematics that you have studied previously: calculus is less static and more dynamic. It is concerned with change and motion; it deals with quantities that approach other quantities. For that reason it may be useful to have an overview of the subject before beginning its intensive study. Here we give a glimpse of some of the main ideas of calculus by showing how the concept of a limit arises when we attempt to solve a variety of problems.

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.



$A = A_1 + A_2 + A_3 + A_4 + A_5$

FIGURE 1

The Area Problem

The origins of calculus go back at least 2500 years to the ancient Greeks, who found areas using the "method of exhaustion." They knew how to find the area A of any polygon by dividing it into triangles as in Figure 1 and adding the areas of these triangles.

It is a much more difficult problem to find the area of a curved figure. The Greek method of exhaustion was to inscribe polygons in the figure and circumscribe polygons about the figure and then let the number of sides of the polygons increase. Figure 2 illustrates this process for the special case of a circle with inscribed regular polygons.

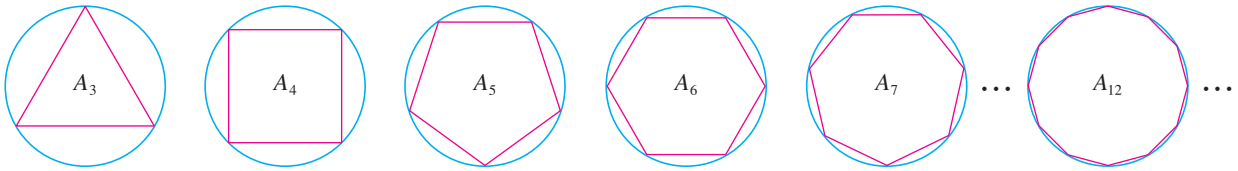


FIGURE 2

TEC In the Preview Visual, you can see how areas of inscribed and circumscribed polygons approximate the area of a circle.

Let A_n be the area of the inscribed polygon with n sides. As n increases, it appears that A_n becomes closer and closer to the area of the circle. We say that the area of the circle is the limit of the areas of the inscribed polygons, and we write

$$A = \lim_{n \to \infty} A_n$$

The Greeks themselves did not use limits explicitly. However, by indirect reasoning, Eudoxus (fifth century BC) used exhaustion to prove the familiar formula for the area of a circle: $A = \pi r^2$.

We will use a similar idea in Chapter 5 to find areas of regions of the type shown in Figure 3. We will approximate the desired area A by areas of rectangles (as in Figure 4), let the width of the rectangles decrease, and then calculate A as the limit of these sums of areas of rectangles.

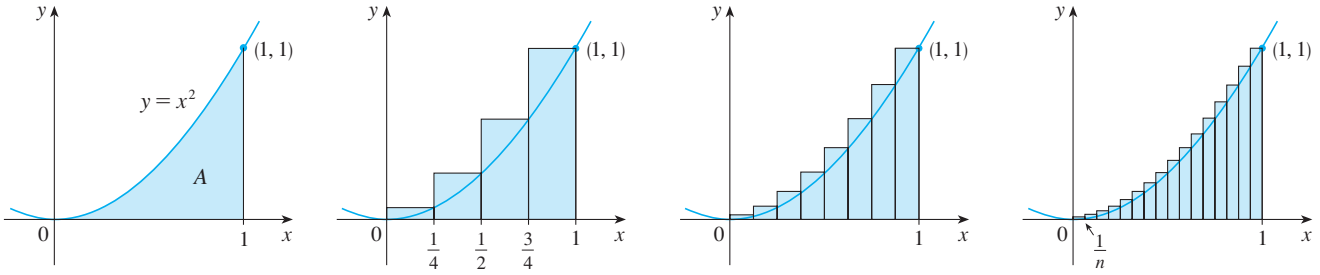


FIGURE 3

The area problem is the central problem in the branch of calculus called *integral calculus*. The techniques that we will develop in Chapter 5 for finding areas will also enable us to compute the volume of a solid, the length of a curve, the force of water against a dam, the mass and center of gravity of a rod, and the work done in pumping water out of a tank.

The Tangent Problem

Consider the problem of trying to find an equation of the tangent line t to a curve with equation $y = f(x)$ at a given point P. (We will give a precise definition of a tangent line in

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

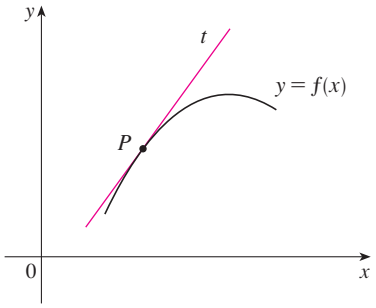


FIGURE 5
The tangent line at P

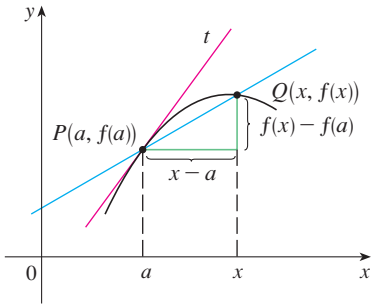


FIGURE 6
The secant line at PQ

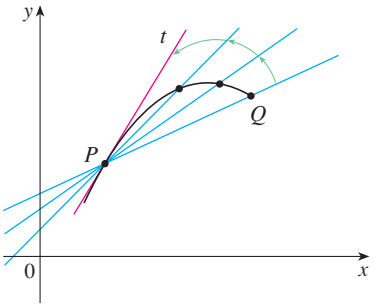


FIGURE 7
Secant lines approaching the tangent line

Chapter 2. For now you can think of it as a line that touches the curve at P as in Figure 5.) Since we know that the point P lies on the tangent line, we can find the equation of t if we know its slope m. The problem is that we need two points to compute the slope and we know only one point, P, on t. To get around the problem we first find an approximation to m by taking a nearby point Q on the curve and computing the slope m_{PQ} of the secant line PQ. From Figure 6 we see that

$$m_{PQ} = \frac{f(x) - f(a)}{x - a} \quad \boxed{1}$$

Now imagine that Q moves along the curve toward P as in Figure 7. You can see that the secant line rotates and approaches the tangent line as its limiting position. This means that the slope m_{PQ} of the secant line becomes closer and closer to the slope m of the tangent line. We write

$$m = \lim_{Q \to P} m_{PQ}$$

and we say that m is the limit of m_{PQ} as Q approaches P along the curve. Because x approaches a as Q approaches P, we could also use Equation 1 to write

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \boxed{2}$$

Specific examples of this procedure will be given in Chapter 2.

The tangent problem has given rise to the branch of calculus called *differential calculus,* which was not invented until more than 2000 years after integral calculus. The main ideas behind differential calculus are due to the French mathematician Pierre Fermat (1601–1665) and were developed by the English mathematicians John Wallis (1616–1703), Isaac Barrow (1630–1677), and Isaac Newton (1642–1727) and the German mathematician Gottfried Leibniz (1646–1716).

The two branches of calculus and their chief problems, the area problem and the tangent problem, appear to be very different, but it turns out that there is a very close connection between them. The tangent problem and the area problem are inverse problems in a sense that will be described in Chapter 5.

Velocity

When we look at the speedometer of a car and read that the car is traveling at 48 km/h, what does that information indicate to us? We know that if the velocity remains constant, then after an hour we will have traveled 48 km. But if the velocity of the car varies, what does it mean to say that the velocity at a given instant is 48 km/h?

In order to analyze this question, let's examine the motion of a car that travels along a straight road and assume that we can measure the distance traveled by the car (in meters) at l-second intervals as in the following chart:

t = Time elapsed (s)	0	1	2	3	4	5
d = Distance (m)	0	2	9	24	42	71

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

As a first step toward finding the velocity after 2 seconds have elapsed, we find the average velocity during the time interval $2 \leqslant t \leqslant 4$:

$$\text{average velocity} = \frac{\text{change in position}}{\text{time elapsed}}$$

$$= \frac{42 - 9}{4 - 2}$$

$$= 16.5 \text{ m/s}$$

Similarly, the average velocity in the time interval $2 \leqslant t \leqslant 3$ is

$$\text{average velocity} = \frac{24 - 9}{3 - 2} = 15 \text{ m/s}$$

We have the feeling that the velocity at the instant $t = 2$ can't be much different from the average velocity during a short time interval starting at $t = 2$. So let's imagine that the distance traveled has been measured at 0.l-second time intervals as in the following chart:

t	2.0	2.1	2.2	2.3	2.4	2.5
d	9.00	10.02	11.16	12.45	13.96	15.80

Then we can compute, for instance, the average velocity over the time interval $[2, 2.5]$:

$$\text{average velocity} = \frac{15.80 - 9.00}{2.5 - 2} = 13.6 \text{ m/s}$$

The results of such calculations are shown in the following chart:

Time interval	[2, 3]	[2, 2.5]	[2, 2.4]	[2, 2.3]	[2, 2.2]	[2, 2.1]
Average velocity (m/s)	15.0	13.6	12.4	11.5	10.8	10.2

The average velocities over successively smaller intervals appear to be getting closer to a number near 10, and so we expect that the velocity at exactly $t = 2$ is about 10 m/s. In Chapter 2 we will define the instantaneous velocity of a moving object as the limiting value of the average velocities over smaller and smaller time intervals.

In Figure 8 we show a graphical representation of the motion of the car by plotting the distance traveled as a function of time. If we write $d = f(t)$, then $f(t)$ is the number of meters traveled after t seconds. The average velocity in the time interval $[2, t]$ is

$$\text{average velocity} = \frac{\text{change in position}}{\text{time elapsed}} = \frac{f(t) - f(2)}{t - 2}$$

which is the same as the slope of the secant line PQ in Figure 8. The velocity v when $t = 2$ is the limiting value of this average velocity as t approaches 2; that is,

$$v = \lim_{t \to 2} \frac{f(t) - f(2)}{t - 2}$$

and we recognize from Equation 2 that this is the same as the slope of the tangent line to the curve at P.

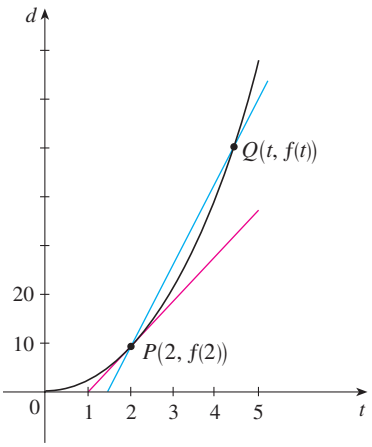


FIGURE 8

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Thus, when we solve the tangent problem in differential calculus, we are also solving problems concerning velocities. The same techniques also enable us to solve problems involving rates of change in all of the natural and social sciences.

The Limit of a Sequence

In the fifth century BC the Greek philosopher Zeno of Elea posed four problems, now known as *Zeno's paradoxes*, that were intended to challenge some of the ideas concerning space and time that were held in his day. Zeno's second paradox concerns a race between the Greek hero Achilles and a tortoise that has been given a head start. Zeno argued, as follows, that Achilles could never pass the tortoise: Suppose that Achilles starts at position a_1 and the tortoise starts at position t_1. (See Figure 9.) When Achilles reaches the point $a_2 = t_1$, the tortoise is farther ahead at position t_2. When Achilles reaches $a_3 = t_2$, the tortoise is at t_3. This process continues indefinitely and so it appears that the tortoise will always be ahead! But this defies common sense.

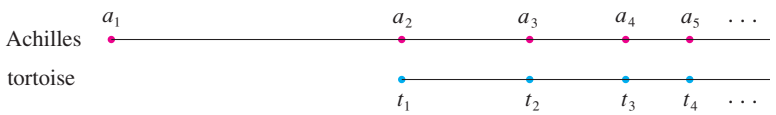


FIGURE 9

One way of explaining this paradox is with the idea of a *sequence*. The successive positions of Achilles $(a_1, a_2, a_3, \ldots)$ or the successive positions of the tortoise $(t_1, t_2, t_3, \ldots)$ form what is known as a sequence.

In general, a sequence $\{a_n\}$ is a set of numbers written in a definite order. For instance, the sequence

$$\left\{1, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \tfrac{1}{5}, \ldots\right\}$$

can be described by giving the following formula for the nth term:

$$a_n = \frac{1}{n}$$

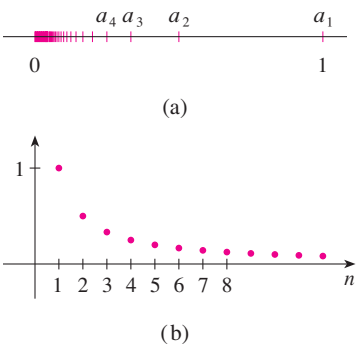


FIGURE 10

We can visualize this sequence by plotting its terms on a number line as in Figure 10(a) or by drawing its graph as in Figure 10(b). Observe from either picture that the terms of the sequence $a_n = 1/n$ are becoming closer and closer to 0 as n increases. In fact, we can find terms as small as we please by making n large enough. We say that the limit of the sequence is 0, and we indicate this by writing

$$\lim_{n \to \infty} \frac{1}{n} = 0$$

In general, the notation

$$\lim_{n \to \infty} a_n = L$$

is used if the terms a_n approach the number L as n becomes large. This means that the numbers a_n can be made as close as we like to the number L by taking n sufficiently large.

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

The concept of the limit of a sequence occurs whenever we use the decimal representation of a real number. For instance, if

$$a_1 = 3.1$$

$$a_2 = 3.14$$

$$a_3 = 3.141$$

$$a_4 = 3.1415$$

$$a_5 = 3.14159$$

$$a_6 = 3.141592$$

$$a_7 = 3.1415926$$

$$\vdots$$

then $$\lim_{n \to \infty} a_n = \pi$$

The terms in this sequence are rational approximations to π.

Let's return to Zeno's paradox. The successive positions of Achilles and the tortoise form sequences $\{a_n\}$ and $\{t_n\}$, where $a_n < t_n$ for all n. It can be shown that both sequences have the same limit:

$$\lim_{n \to \infty} a_n = p = \lim_{n \to \infty} t_n$$

It is precisely at this point p that Achilles overtakes the tortoise.

The Sum of a Series

Another of Zeno's paradoxes, as passed on to us by Aristotle, is the following: "A man standing in a room cannot walk to the wall. In order to do so, he would first have to go half the distance, then half the remaining distance, and then again half of what still remains. This process can always be continued and can never be ended." (See Figure 11.)

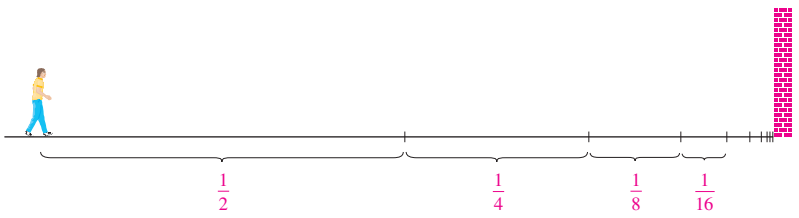


FIGURE 11

Of course, we know that the man can actually reach the wall, so this suggests that perhaps the total distance can be expressed as the sum of infinitely many smaller distances as follows:

3 $$1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{2^n} + \cdots$$

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Zeno was arguing that it doesn't make sense to add infinitely many numbers together. But there are other situations in which we implicitly use infinite sums. For instance, in decimal notation, the symbol $0.\overline{3} = 0.3333\ldots$ means

$$\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10{,}000} + \cdots$$

and so, in some sense, it must be true that

$$\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10{,}000} + \cdots = \frac{1}{3}$$

More generally, if d_n denotes the nth digit in the decimal representation of a number, then

$$0.d_1d_2d_3d_4\ldots = \frac{d_1}{10} + \frac{d_2}{10^2} + \frac{d_3}{10^3} + \cdots + \frac{d_n}{10^n} + \cdots$$

Therefore some infinite sums, or infinite series as they are called, have a meaning. But we must define carefully what the sum of an infinite series is.

Returning to the series in Equation 3, we denote by s_n the sum of the first n terms of the series. Thus

$$s_1 = \tfrac{1}{2} = 0.5$$

$$s_2 = \tfrac{1}{2} + \tfrac{1}{4} = 0.75$$

$$s_3 = \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} = 0.875$$

$$s_4 = \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} = 0.9375$$

$$s_5 = \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \tfrac{1}{32} = 0.96875$$

$$s_6 = \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \tfrac{1}{32} + \tfrac{1}{64} = 0.984375$$

$$s_7 = \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \tfrac{1}{32} + \tfrac{1}{64} + \tfrac{1}{128} = 0.9921875$$

$$\vdots$$

$$s_{10} = \tfrac{1}{2} + \tfrac{1}{4} + \cdots + \tfrac{1}{1024} \approx 0.99902344$$

$$\vdots$$

$$s_{16} = \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{16}} \approx 0.99998474$$

Observe that as we add more and more terms, the partial sums become closer and closer to 1. In fact, it can be shown that by taking n large enough (that is, by adding sufficiently many terms of the series), we can make the partial sum s_n as close as we please to the number 1. It therefore seems reasonable to say that the sum of the infinite series is 1 and to write

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} + \cdots = 1$$

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

In other words, the reason the sum of the series is 1 is that

$$\lim_{n \to \infty} s_n = 1$$

In Chapter 11 we will discuss these ideas further. We will then use Newton's idea of combining infinite series with differential and integral calculus.

Summary

We have seen that the concept of a limit arises in trying to find the area of a region, the slope of a tangent to a curve, the velocity of a car, or the sum of an infinite series. In each case the common theme is the calculation of a quantity as the limit of other, easily calculated quantities. It is this basic idea of a limit that sets calculus apart from other areas of mathematics. In fact, we could define calculus as the part of mathematics that deals with limits.

After Sir Isaac Newton invented his version of calculus, he used it to explain the motion of the planets around the sun. Today calculus is used in calculating the orbits of satellites and spacecraft, in predicting population sizes, in estimating how fast oil prices rise or fall, in forecasting weather, in measuring the cardiac output of the heart, in calculating life insurance premiums, and in a great variety of other areas. We will explore some of these uses of calculus in this book.

In order to convey a sense of the power of the subject, we end this preview with a list of some of the questions that you will be able to answer using calculus:

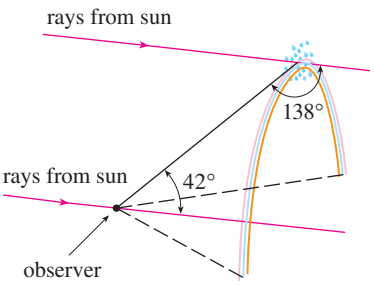


FIGURE 12

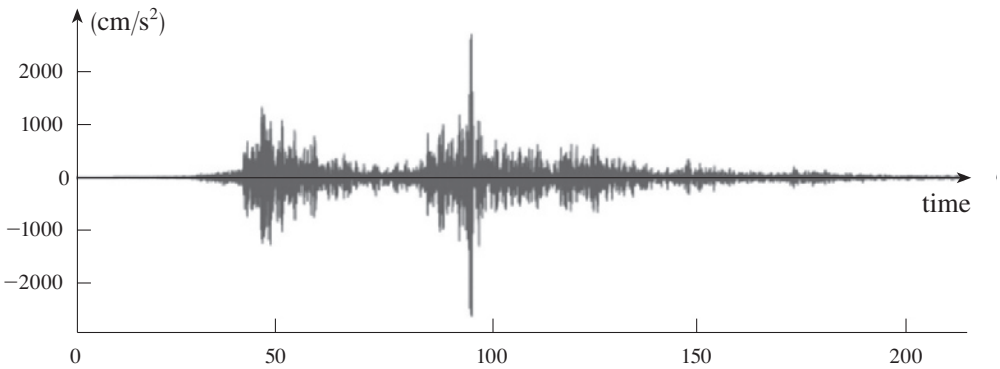
1. How can we explain the fact, illustrated in Figure 12, that the angle of elevation from an observer up to the highest point in a rainbow is 42°? (See page 285.)
2. How can we explain the shapes of cans on supermarket shelves? (See page 343.)
3. Where is the best place to sit in a movie theater? (See page 465.)
4. How can we design a roller coaster for a smooth ride? (See page 182.)
5. How far away from an airport should a pilot start descent? (See page 208.)
6. How can we fit curves together to design shapes to represent letters on a laser printer? (See page 657.)
7. How can we estimate the number of workers that were needed to build the Great Pyramid of Khufu in ancient Egypt? (See page 460.)
8. Where should an infielder position himself to catch a baseball thrown by an outfielder and relay it to home plate? (See page 465.)
9. Does a ball thrown upward take longer to reach its maximum height or to fall back to its original height? (See page 609.)
10. How can we explain the fact that planets and satellites move in elliptical orbits? (See page 876.)
11. How can we distribute water flow among turbines at a hydroelectric station so as to maximize the total energy production? (See page 980.)
12. If a marble, a squash ball, a steel bar, and a lead pipe roll down a slope, which of them reaches the bottom first? (See page 1052.)

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

1 Functions and Models

Pictura Collectus/Alamy

Often a graph is the best way to represent a function because it conveys so much information at a glance. Shown is a graph of the vertical ground acceleration created by the 2011 earthquake near Tohoku, Japan. The earthquake had a magnitude of 9.0 on the Richter scale and was so powerful that it moved northern Japan 2.4 meters closer to North America.



Seismological Society of America

THE FUNDAMENTAL OBJECTS THAT WE deal with in calculus are functions. This chapter prepares the way for calculus by discussing the basic ideas concerning functions, their graphs, and ways of transforming and combining them. We stress that a function can be represented in different ways: by an equation, in a table, by a graph, or in words. We look at the main types of functions that occur in calculus and describe the process of using these functions as mathematical models of real-world phenomena.

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.